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On the spectrum behaviour of vibrated granular matter

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Abstract

A laser facility based on a linear image sensor with a sampling period of 100 μs allows the investigation of the dissipative dynamics of a vibrated granular matter under gravity. The laser reveals the vertical movement of an individual zirconia–yttria stabilized 2 mm ball at the surface of a weakly excited 3D granular matter bed. The stochastic realizations are measured from the top of the container. Then, power spectra measurements reveal the different cooperative dynamics of the fluidized gap. We also carried out measurements for one steel ball and many balls in 1D and 3D systems. We fit the measured different regimes with generalized Langevin pictures. We introduce a fractional temporal operator to characterize the ensemble of dissipative particles which cannot be represented by a *single* Langevin particle in a complex fluid.

1. Introduction

Granular media (GM) are ubiquitous in nature and exhibit a wealth of intriguing physical properties [1]. In this paper we study the correlations and the statistical properties of vibrated GM under gravity. Since energy is constantly being added to the system, a non-equilibrium steady state is reached [2, 3]. We show that in the weakly excited regime the dynamical behaviour of the fluidized particles cannot be described as *simple* Brownian particles; this fact leads us to the conclusions that the cooperative dissipative dynamics of the GM particles can be described in terms of generalized Langevin particles.

Hayakawa and Hong [4] introduced the thermodynamics approach for weakly excited GM. They studied vibrated GM under gravity, and mapped the non-equilibrium system with a spinless Fermion-like theory. The important point is that by focusing on the configurational properties of an excluded volume (hole) theory, the GM non-equilibrium steady state can be understood in terms of a configurational (maximum principle) Fermi-like distribution.

Many interesting questions concerning the complex system of GM are still open, i.e., *the* stochastic motion $z(t)$ of the fluidized particles is not entirely known [5]. For example it

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is important to test whether the realization spectrum, $S_z(f)$, behaves as a Brownian motion ($1/f^2$), or has a more complex behaviour related to a cooperative dynamics. Therefore an analysis of the stochastic realization $z(t)$ of these *macroscopic* Fermi-like particles (*m*Fp) should be made. In this paper we report some results concerning the analysis of these *m*Fp; in particular we have measured these realizations $z(t)$ and from their Fourier (FFT) analysis we conclude that $S_z(f) \sim f^{-\nu}$ with $\nu = \nu(f_e, \Gamma)$, (where $\Gamma = A\omega^2/g$, A is the amplitude and $\omega = 2\pi f_e$ the frequency of the bed oscillations), showing an agreement with our phenomenological (Langevin) memory-like stochastic differential equation (SDE) approach.

A sinusoidal vibration is driven by a vibration plate on the GM bed. The vibration apparatus is set up by an electromagnetic shaker [TIRAVIB 5212], which allows for feedback through a piezoelectric accelerometer (PCB) [6]. This allows us the control of the frequency f_e and the acceleration in the ranges 10–7000 Hz and $2g$–$40g$, respectively, where g is the acceleration of gravity. The control loop is completed by an Oscillator Lab-works SC121 and a TIRA $19/z$ amplifier of 1 kW.

The 13 layers of GM were set up with ZrO_2–Y_2O_3 balls of diameter $d = 1.99$ mm in a glass container of 30 mm of diameter with steel bottom; see figure 1(a). Also experiments with steel balls of $d = 10.7$ mm in different glass cylinders (11.2 and 11.4 mm) with steel bottoms were carried out.

In all cases, the stochastic realizations $z(t)$ in the direction of the acceleration of gravity were measured from the top of the cylindrical containers, during the vibration under gravity and at a given f_e. The experiments were carried out in a chamber at 1 atm of air with 5.8 ± 0.2 g m^{-3} of water vapour. The absolute humidity was controlled by using a Peltier condenser and a control loop through a thermo-hygrometer. In the case of the experiment with ZrO_2–Y_2O_3 balls, the humidity is of major relevance in order to control the particle–particle and particle–wall contact forces [7, 8]. Under these humidity controlled conditions, there was not observed any surface convection, convection rolls in the GM bed, nor rotational movement of the bed with respect to the container, which is typical for a content of water vapour >10 g m^{-3}.

The realization $z(t)$ of one particle was followed in a 1D window of 12 mm with a laser device by using a triangulation method; see figure 1(a). In figure 1(b) we show a typical realization $z(t)$ when one steel ball is vibrated at $f_e = 250$ Hz with $\Gamma = 20$; under such experimental conditions (sinusoidal excitation amplitude $A = \Gamma g/\omega^2 \sim 79.6$ μm) it is possible to observe the quasi-non-erratic parabolas for the movement of the ball under gravity.

A laser emitter with a spot of 70 μm and a linear image sensor (CCD-like array) enables a high speed measurement with 100 μs sampling. The linear image sensing method measures the peak position values for the light spots and suppresses the perturbation of secondary peaks, which makes possible a resolution of 1 μm. The shaker and the laser displacement sensor were placed on vibration-isolated tables to isolate them from external vibrations, and the displacement sensor from the experiment vibrations. The realization $z(t)$ is a measure of the oscillations of the distance (difference) between the particle and the sensor, around the surface of the granular bed (the fluidized 3D gap). The measured realization without excitation reveals a white noise <10 μm. Then due to the experimental set-up our effective resolution is no higher than 10 μm.

The two-second long register of $z(t)$ and the FFT analysis with Nyquist frequencies of 0.5 and 1.25 kHz were taken with a 9354 C Le Croy Oscilloscope operating at 500 MHz. The velocity $V(t) = \mathrm{d}z/\mathrm{d}t$ of the *m*Fp and its square dispersion $\sigma_V^2 = \langle V(t)^2\rangle - \langle V(t)\rangle^2$ were calculated from the $z(t)$ registers. In figure 2(a) we report σ_V^2 against $\sigma_z = \sqrt{\langle z(t)^2\rangle - \langle z(t)\rangle^2}$ for fixed $\Gamma = 20$ and several f_e from 200 to 340 Hz for one steel ball experiments (circles and squares), and also from 250 to 400 Hz for nine steel balls in a column.

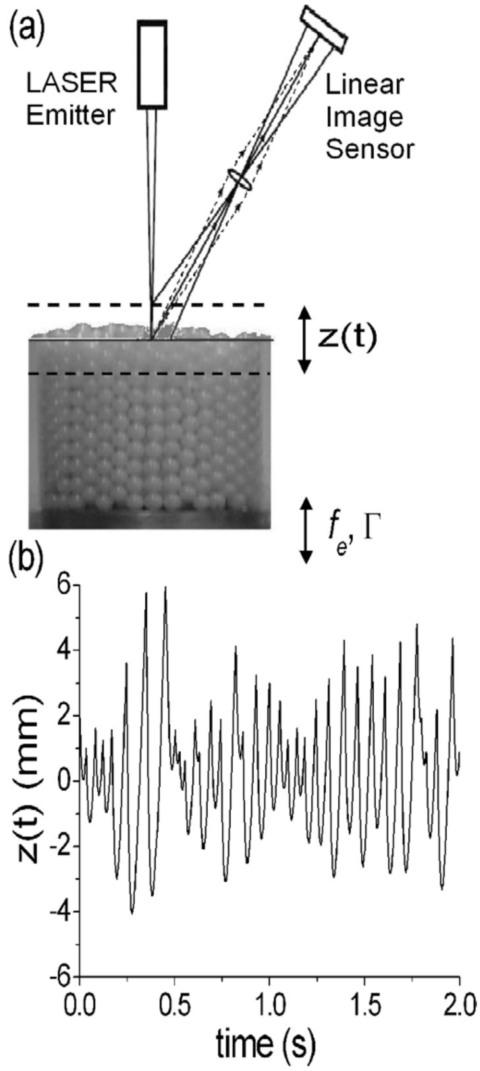

Figure 1. (a) Geometry of the distance measurement (realizations of the amplitude $z(t)$) corresponding to 13 layers of GM with $d = 1.99$ mm ZrO_2–Y_2O_3 balls in a glass container of 30 mm of diameter. (b) A typical realization $z(t)$ for one steel ball ($d = 10.7$ mm) in a 1D column, when it is performing a quasi-non-erratic movement under gravity is shown. In this case $f_e = 250$ Hz and $\Gamma = 20$, thus corresponding to the relative length $A/d \sim 0.0074$ and dimensionless maximum velocity $V_b = A\omega/\sqrt{gd} \sim 0.39$.

The experiments with many ZrO_2–Y_2O_3 balls ($d = 1.99$ mm) were carried out for fixed $\Gamma = 20$ and f_e from 70 to 600 Hz; thus the GM bed was driven by an excitation of amplitude A from 1 mm to 13 μm. In addition, one minute films of this fluidized 3D gap were taking by using a video camera microscope (CVM). For the propose of this work the CVM was only used to identify qualitatively the different dynamical regimes of the ZrO_2–Y_2O_3 balls; in fact the recorded movies were only used to discuss the physical meaning of the measured $z(t)$ registers and their corresponding realization spectra $S_z(f)$. These measurements at $\Gamma = 20$ are a partial research of a large work in progress we carried out with $5 < \Gamma < 40$.

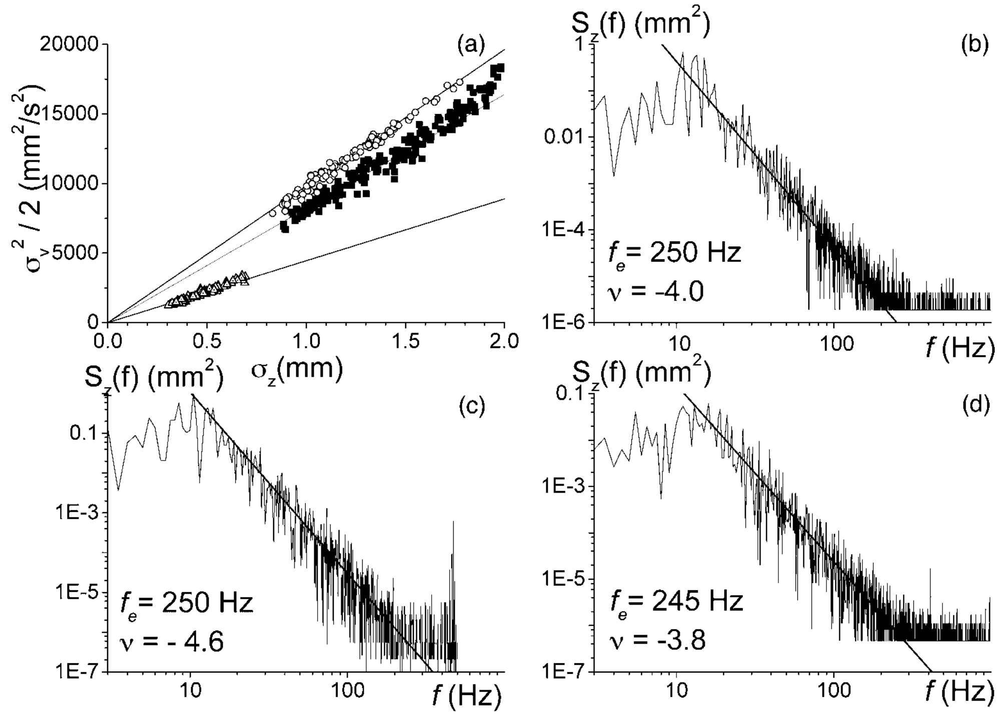

Figure 2. (a) σ_V^2 as a function of σ_z. The circles correspond to one steel ball, full-squares to one steel ball with rotation, and triangles to nine steel balls in a 1D array. Typical log–log power spectrum for one steel ball (b) and (c) and nine steel balls (d) in a 1D array.

2. The spectrum of the realizations

When the input of energy is large the individual motion of the beads looks erratic. In this section we will analyse the frequency spectrum of the stochastic realizations $z(t)$ of the mFp at the fluidized gap. We remark that in this report we will not be concerned about the regime where the input of energy is so small that the power spectrum can be described by a Feigenbaun scenario [3].

Consider a given stochastic process (SP) $u(t)$. Then under quite general conditions if the characteristic function of the SP $u(t)$ (Fourier transform of the probability distribution $G_u(k, t) = \mathcal{F}_k[P(u, t)]$) obeys the *asymptotic* scaling [9, 10]:

$$G_u\left(\frac{k}{\Lambda^H}, \Lambda t\right) \to G_u(k, t), \tag{1}$$

which means that the SP $u(t)$ fulfils in distribution the scaling $\Lambda^{-H} u(\Lambda t) \to u(t)$, then the spectrum density behaves asymptotically like

$$S_u(f) \propto 1/f^{2H+1}.$$

Now let us apply this result to a Langevin-like particle. Consider the case when the velocity $V(t)$ and the position $z(t)$ of the particle are described by the SDE

$$\frac{\mathrm{d}V(t)}{\mathrm{d}t} = -\gamma V(t) + \xi(t), \qquad \text{and} \qquad \frac{\mathrm{d}z}{\mathrm{d}t} = V(t), \tag{2}$$

when $\xi(t)$ is a Gaussian coloured noise. This SDE corresponds to the alternative treatment of Brownian motion, in fact initiated by Langevin [12, 10]. This coloured noise case corresponds to a weak non-Markovian process [11]. From (2) and (1) we obtain, in the overdamped case (Brownian motion), $S_z(f) \propto 1/f^2$ (limit $f \ll \gamma$). In contrast, in the undamped free motion case (random accelerations model) we obtain $S_z(f) \propto 1/f^4$ (limit $f \gg \gamma$). Nevertheless, in the case when the correlation of the noise $\xi(t)$ in (2) is of the *long-range* type $\langle \xi(t)\xi(0)\rangle \propto t^{-\theta}$ with $0 < \theta < 1$ (perhaps describing a complex fluid surrounding our *test* particle), the SP $z(t)$ turns out to be a *strong* non-Markovian process [11], i.e., there is no Fokker–Planck asymptotic regime. Then we get for the spectrum in the overdamped case

$$S_z(f) \propto 1/f^{3-\theta}, \qquad \text{if } \mathrm{d}z/\mathrm{d}t = \xi(t), \quad \text{with } 0 < \theta < 1, \tag{3}$$

and in the undamped case

$$S_z(f) \propto 1/f^{5-\theta}, \qquad \text{if } \mathrm{d}^2z/\mathrm{d}t^2 = \xi(t), \quad \text{with } 0 < \theta < 1. \tag{4}$$

For the particular situation when the ensemble of mFp cannot be represented by a *single* particle in a ‘fluid’ (Langevin-like particle) we should leave the regular stochastic calculus and introduce a *fractional* temporal differential operator to emulate the complex multiple particle collisions and particle motion. Then we model the GM system by a fractional Langevin equation of the form [13]

$$ {}_0D_t^{\alpha}[V(t)] - V_0\frac{t^{-\alpha}}{\Gamma(1-\alpha)} = -\gamma^{\alpha}V(t) + \xi(t), \qquad \text{and} \qquad \frac{\mathrm{d}z}{\mathrm{d}t} = V(t), \quad \alpha > 0, \tag{5}$$

where $\xi(t)$ is a Gaussian white noise. So, asymptotically in the undamped limit, $f \gg \gamma$, we get

$$G_V\left(\frac{k}{\Lambda^H}, \Lambda t\right) \to G_V(k,t), \qquad \text{with } H = \tfrac{1}{2} - (1-\alpha), \quad \text{when } \alpha \in \left(\tfrac{1}{2}, 1\right), \tag{6}$$

which means that $S_V(f) \propto 1/f^{2\alpha}$. Then using $\mathrm{d}z/\mathrm{d}t = V(t)$ it follows that

$$S_z(f) = \frac{1}{f^2}S_V(f) \propto 1/f^{2(1+\alpha)}, \qquad \alpha \in \left(\tfrac{1}{2}, 1\right), \tag{7}$$

which is the desired result to fit our experiments when friction and collisions between the GM particles are important issues, and cannot be represented by a memory-like Brownian model as in equation (3), nor by using the memory-like random acceleration model as in equation (4).

The general case given in equation (5) by keeping arbitrary the friction coefficient γ and for any noise $\xi(t)$ and correlation $\langle \xi(t)\xi(0)\rangle$ can also be solved in a similar way [13].

3. Results and discussion

In figure 2(a) we show the velocity squared dispersion σ_V^2 against the amplitude-dispersion σ_z of the realizations of the mFp corresponding to the 1D array of 10.7 mm steel balls vibrated vertically; for weak amplitude the slope σ_V^2/σ_z shows a linear behaviour. Figure 2(a) shows three experiments. The first corresponds to *one steel ball* vertically jumping in a cylinder of 11.2 mm (almost elastic case) with $S_z(f) \sim 1/f^4$, see figure 2(b), and gives the slope 9.7 ± 3 m s^{-2} $\sim g$ (in accord with the conservation of energy: $mg\sigma_z = \frac{1}{2}m\sigma_V^2$). The second experiment is *one steel ball* jumping in a wider cylinder (11.4 mm); in this case the slope gives 8.2 ± 3 m s^{-2} indicating a dissipative mechanism, here typically $S_z(f) \sim 1/f^{4.6}$; see figure 2(c). The wider cylinder permits the rotation of the steel ball, probably due to the interaction with the air gap between the steel ball and the wall. In the third experiment there are *nine steel balls* in the wider cylinder; in this case the many-body collisions came into account giving a smaller slope 5.9 ± 3 m s^{-2}, here $S_z(f) \sim 1/f^{3.8}$; see figure 2(d) (we find that for 400 Hz $> f_e >$ 200 Hz, the exponent is $3.5 < \nu < 3.8$). This last result indicates

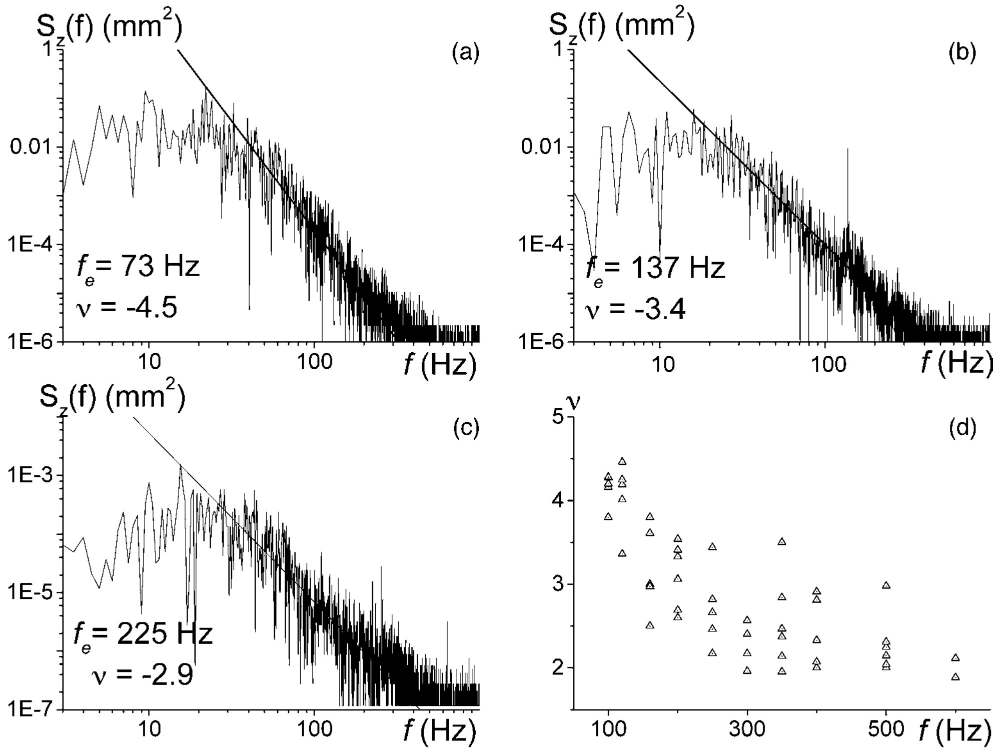

Figure 3. Log–log power spectrum for 13 layers of ZrO_2–Y_2O_3 balls in a glass container as in figure 1(a), for $\Gamma = 20$ and different excitation frequency f_e, (a) 73 Hz, (b) 137 Hz, (c) 225 Hz. (d) The power spectrum exponent ν against the excitation frequency f_e is shown.

that in this regime it is necessary to introduce a description in terms of a non-usual collision operator. The estimated slope σ_V^2/σ_z gives a qualitative magnitude from which it is possible to observe a clear tendency in the GM motion behaviour.

Spectral density measurements (corresponding to the geometry of figure 1(a)) associated to the realization $z(t)$ of ZrO_2–Y_2O_3 balls are shown in figures 3(a)–(c). In figure 3(d) the plot of the exponent ν against the excitation frequency f_e for a fixed acceleration $\Gamma = 20$ is shown. For $f_e = 600$ Hz, the balls are organized in a 'lattice' (we test this fact by using a VCM at the surface of the GM bed) where the weak-vibration movement of each ball is limited to a site of this lattice, given therefore a Wiener realization $z(t)$. Note that for $\Gamma = 20$ and $f_e = 600$ Hz the excitation amplitude is $A \sim 13.8$ μm, so we do not expect *hopping motion* from one site to another. It is interesting to remark that even when the values $A/d \sim 0.007$ and $A\omega/\sqrt{gd} \sim 0.37$ for the zirconia–yttria balls are similar to the one from the experiment with only one steel ball (for $\Gamma = 20$ and $f_e = 250$ Hz), in the present case the stochastic behaviour is quite different, as can be corroborated from the value of $\nu \sim 2$, when it is compared with the one from figures 2(b) and (c).

For f_e approximately between 500 and 200 Hz, the 'hopping' of the fluidized balls (jumping to unoccupied hole-states) are more frequent, thus leading to exponents ν from 2 to 3. This is a generalized Brownian motion from low to high correlation; see equation (3). When $\nu = 3$ is reached we observe a pure 'hopping' mechanism and this is related to a highly non-Markovian memory in the SDE (limit $\theta \to 0$), i.e., the long hopping of one ball is associated with the long-range noise correlation of the 'fluid'. Such a fluid should be interpreted as the hopping 'events' of the whole granular matter bed. For exponents ν between 3 and 4 (if $f_e \approx 200$–150 Hz), the motion of a single particle evolves from a pure hopping to a more complex behaviour where the hopping balls are perturbed by the collisions from the many-body bed. This makes necessary the introduction of a new collision operator which is written in terms of the fractional SDE (5). In other words, the hopping of a ball is perturbed (mainly) by few-body collisions. At frequencies f_e around 70 Hz, we observe a pure collision behaviour where a ball is under the random acceleration mechanism transferred from the surrounding 'fluid', as is proved because ν is between 4 and 5, corresponding to the random acceleration dynamics emulated by equation (4).

In general our SDE picture allows us also to calculate the velocity distribution of the GM; work along this line will be reported elsewhere.

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